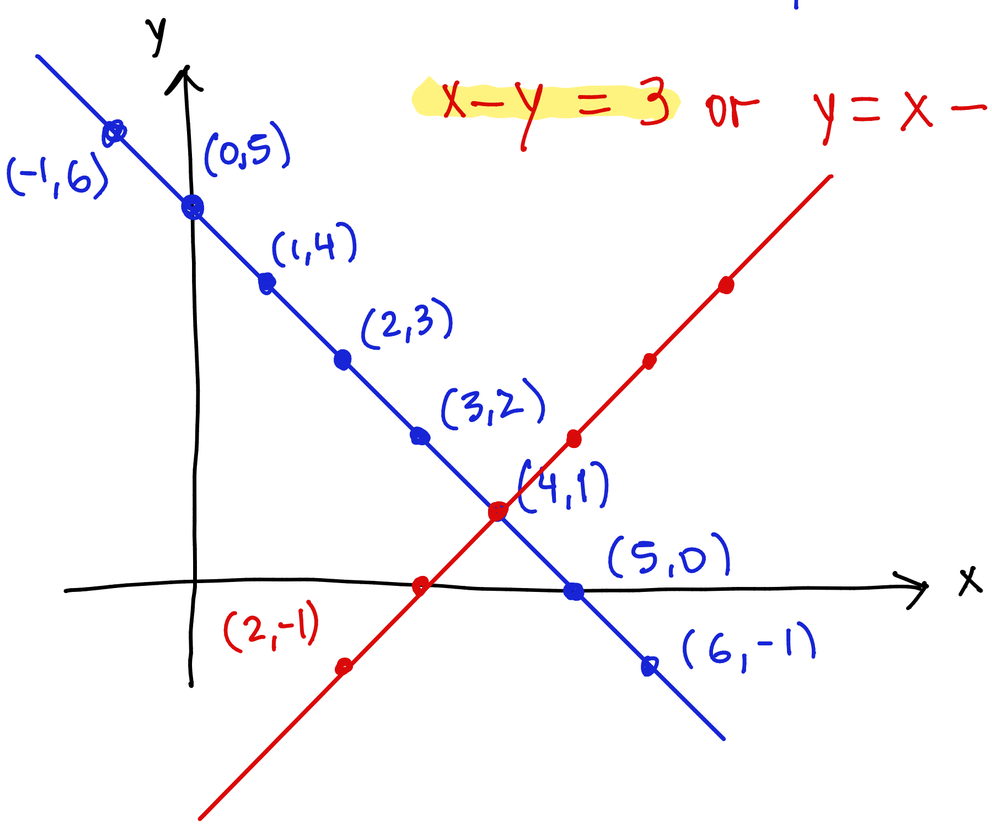
Class 22 - § 12.1 System of Linear Equations

Ex: $3x + 5 = 11 \Rightarrow \boxed{x = 2}$ ← ONE solution

Ex: $x + y = 5 \Rightarrow (x, y)$ $\quad (3,2), (2,3), (4,1), (1,4)$

the sum of two numbers equals five.



$x - y = 3$ or $y = x - 3$

$x + y = 5 \rightarrow y = -x + 5$

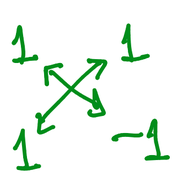
each point on the line $y = -x + 5$ corresponds to a solution of the equation $x + y = 5$

Intersection of two lines is a pair (x, y) that is a solution for both equations

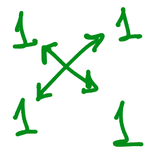
$$\begin{cases} x + y = 5 \\ x - y = 3 \end{cases} \rightarrow$$

ONE solution: $\boxed{(4,1)}$

two equations & two variables

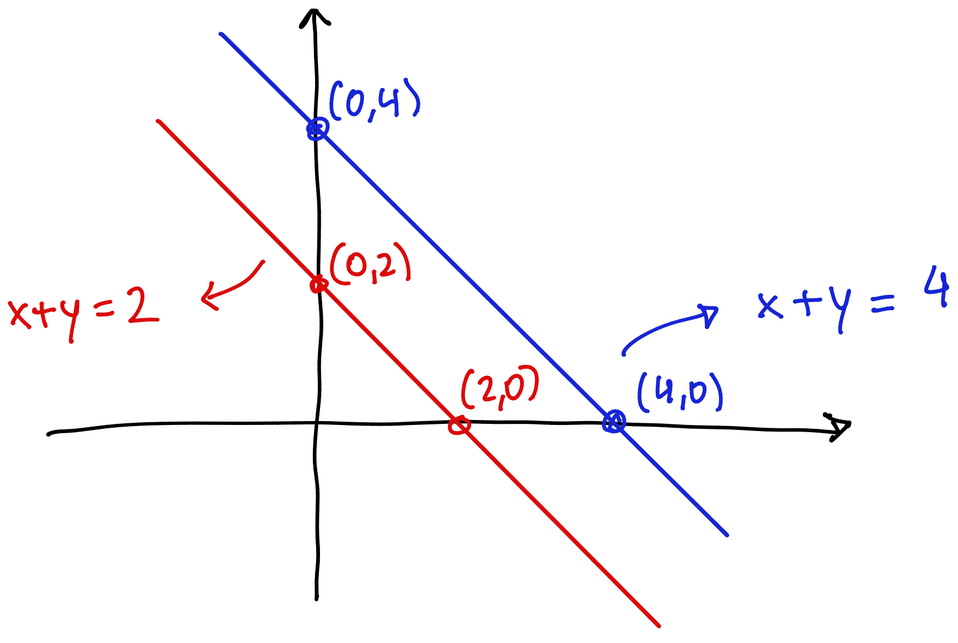


ONE SOLUTION



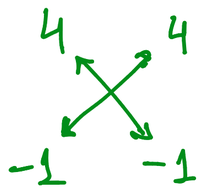
$$\begin{cases} x + y = 4 \\ x + y = 2 \end{cases}$$

NO SOLUTION



Paralled : No solutions

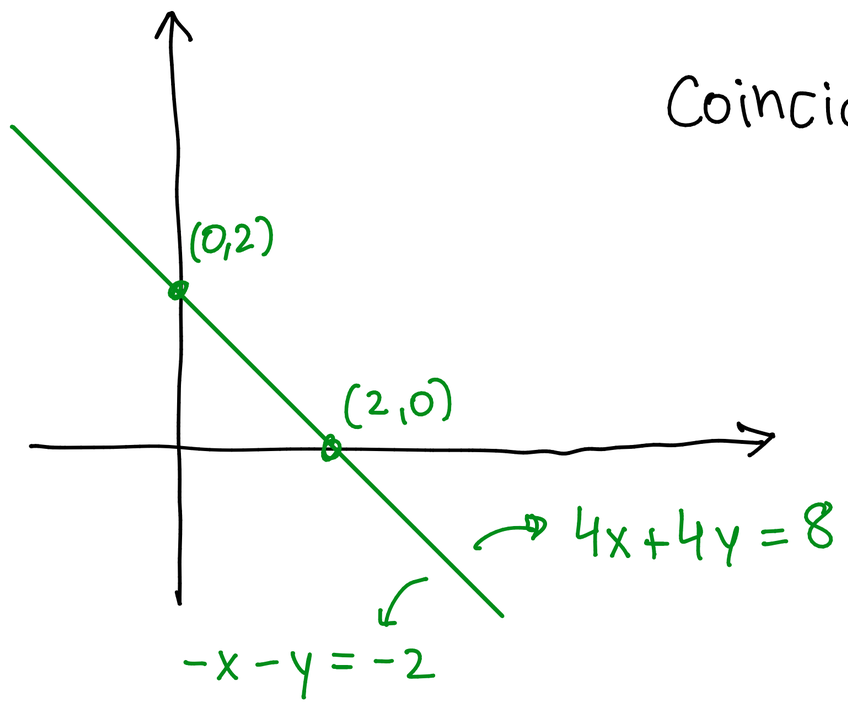
$$\begin{cases} 4x+4y=8 \\ -x-y=-2 \end{cases}$$



NO solutions

OR

∞ solutions



Coincident : infinitely many solutions

Word Phrases ⟷ Algebraic Expressions

$$\begin{cases} x+y=5 \rightarrow \text{the sum of two numbers equals 5.} \\ x-y=3 \rightarrow \text{the diff. of two numbers equals 3.} \end{cases}$$

Ex1: Eight more than four times a number. (x)

+8 ; 4x

4x+8 or 8+4x

Ex2: 26 times a number, increased by 16.5 (y)

26y ; +16.5

26y+16.5 or 16.5+26y

Ex3: Suppose each notebook costs $ 2.45 and a pack of pens costs $ 3.75. If x is the number of notebooks and y is the number of packs of pens. Write an expression that corresponds to buying both of these items.

x: # of Notebooks | units · price | 2.45x + 3.75y

y: # of pack w/ pens

Solving a System of Equations

Ex 1: $\begin{cases} x+4y=6 & ① \\ 2x-y=-6 & ② \end{cases}$

substitution method

1st: choose eqn & solve for a variable: ① → x

$x+4y=6 \rightarrow x=6-4y$

$-4y \quad -4y$

2nd: take the eqn in step 1 and plug in for the other equation:

$x=6-4y \longrightarrow 2x-y=-6 \rightarrow 2(6-4y)-y=-6$

3rd: solve linear eqn:

$12-8y-y=-6 \rightarrow 12-9y=-6 \rightarrow \frac{-9y}{-9}=\frac{-18}{-9} \rightarrow \boxed{y=2}$

$-12 \quad -12$

4th: find the other variable

$x+4y=6 \rightarrow x+4\cdot 2=6 \rightarrow x+8=6 \rightarrow \boxed{x=-2}$

$2x-y=-6 \rightarrow 2x-2=-6 \rightarrow 2x=-4 \rightarrow \boxed{x=-2}$

$x=6-4y \longrightarrow x=6-4\cdot 2 \rightarrow x=6-8 \rightarrow \boxed{x=-2}$

Ex 2: $\begin{cases} 5x-y=27 \\ 4x+6y=8 \end{cases}$

(x, y) = (5, −2)

1st: $5x-y=27 \rightarrow 5x-27=y$

$-27+y \quad -27+y$

2nd: $4x+6y=8 \longrightarrow 4x+6(5x-27)=8$

3rd: $4x+30x-162=8 \rightarrow \frac{34x}{34}=\frac{170}{34} \rightarrow \boxed{x=5}$

$+162 \quad +162$

4th:

$y=5x-27$

$y=5\cdot 5-27$

$\boxed{y=-2}$

Elimination Method

Ex 1:

$$\begin{cases} x+y=8 \\ 2x-y=7 \end{cases} \quad +$$

$$3x = 15$$

$$\boxed{x=5} \quad \boxed{y=3}$$

Ex 2:

$$\begin{cases} 9x-8y=93 \\ 8x+72y=-392 \end{cases}$$

→ Multiply 1st eqn by 9

$$+ \begin{cases} 81x-72y=837 \\ 8x+72y=-392 \end{cases}$$

$$89x = 445$$

$$\boxed{x=5}$$

Find y:

$8x+72y=-392$, $x=5$

$8\cdot 5+72y=-392$

$40+72y=-392$

$-40 \qquad -40$

$\frac{72y}{72} = \frac{-432}{72}$

$$\boxed{y=6}$$

Ex 3:

$$\begin{cases} 3x-5y=10 \\ 2x-4y=0 \end{cases} \quad \begin{matrix} \times(-2) \rightarrow \\ \rightarrow \times 3 \end{matrix}$$

$$+ \begin{cases} -6x+10y=-20 \\ 6x-12y=0 \end{cases}$$

$$-2y = -20$$

$\frac{-2y}{-2} = \frac{-20}{-2}$

$$\boxed{y=10}$$

$y=10$: $2x-4y=0$

$2x-4\cdot 10=0$ → $2x-40=0$

$2x=40$

$$\boxed{x=20}$$

Solving Applied Problems

Ex 1: A parking lot for trucks has 18 wheelers and 22 wheelers. Knowing that the total amount of wheels in the parking lot is 1114, and the total number of vehicles is 55, how many 18 wheelers and 22 wheelers are there?

x: # of 18-wheelers → eqn for # of wheels: $18x+22y=1114$

y: # of 22-wheelers → eqn for # of vehicles: $x+y=55$

$$\begin{cases} 18x+22y=1114 \\ x+y=55 \end{cases}$$

Substitution method: $\boxed{y=55-x}$

$18x+22y=1114 \longrightarrow 18x+22(55-x)=1114$

$18x+1210-22x=1114 \longrightarrow -4x=-96 \longrightarrow \boxed{x=24}$

$-1210 \qquad -1210 \qquad -4 \qquad -4$

$y=55-x \longrightarrow y=55-24 \longrightarrow \boxed{y=31}$

Ex 2: Gaby's piggy bank has nickels and dimes, she counted a total of $5.65 among 78 coins. How many nickels and dimes she has?

1st: choose a letter for each variable.

2nd: write two equations using two variables.

3rd: find the value of the variables.

	# of dimes	Money value
1 dime	1	$0.10 = 1 · .1
5 dimes	5	$0.50 = 5 · .1
10 dimes	10	$1.00 = 10 · .1
25 dimes	25	$2.50 = 25 · .1

1st: d = # of dimes ($0.10) & n = # of nickels ($0.05)

2nd: ① the # of coins: $d+n=78$

② the total money: $0.1d+0.05n=5.65$

$$\begin{cases} d+n=78 \\ 0.1d+0.05n=5.65 \end{cases} \xrightarrow{\times(-20)} + \begin{cases} d+n=78 \\ -2d-n=-113 \end{cases}$$

$$-d = -35 \rightarrow \boxed{d=35}$$

$$d+n=78 \rightarrow \underset{-35}{35}+n=\underset{-35}{78} \rightarrow \boxed{n=33}$$

Ex3: A real state company sells 3-bedroom apartments and 4-bedroom apartments. Consider that the number of bedrooms is 625 distributed among 176 apartments. How many 3-bedroom and 4-bedroom apartments are there?

x: # of 3-bedroom apartments.

y: # of 4-bedroom apartments.

eqn 1 - # of apt's: $\underbrace{\#\text{ of 3-bedroom apt's}}_{x} + \underbrace{\#\text{ 4-bedroom apt's}}_{y}$ is 176

eqn 1 → $x+y=176$

eqn 2 - # of rooms:

$$\left[\begin{array}{l} 4 \text{ bedrooms for each } \overbrace{\text{4-bedroom apt's}}^{y} \\ + \\ 3 \text{ bedrooms for each } \underbrace{\text{3-bedroom apt's}}_{x} \end{array} = 625\right]$$

eqn 2 → $3x+4y=625$

$$\begin{cases} x+y=176 \\ 3x+4y=625 \end{cases} \xrightarrow{x(-3)} \begin{cases} -3x-3y=-528 \\ +\ 3x+4y=625 \end{cases}$$

$$\boxed{y=97}$$

$$x+y=176 \rightarrow x+97=176 \rightarrow \boxed{x=79}$$
$$\quad\ -97 \quad -97$$

Ex 4: Ned, the owner of Ned's Nut Shop, sells peanuts for $10 per pound and cashews for $11 per pound. Ned wants to create a 20 pound barrel of mixed nuts and sell it for $10.35 per pound. How many pounds of cashews and peanuts shoul Ned use?

x : pounds of cashews & y : pounds of peanuts

$$\begin{cases} x + y = 20 \quad (1) \\ 11x + 10y = 207 \quad (2) \end{cases} \rightarrow \boxed{x = 7} \quad \boxed{y = 13}$$